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TWO-DIMENSIONAL SURFACES IN EUCLIDEAN 5-SPACE WITH CONSTANT SCALAR CURVATURE

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ABSTRACT

In this paper, we analyzed the problem of studying locally the scalar curvature S of the three dimensional surfaces foliated by an equiform motion of catenary curve in Euclidian five spaceE^5. We express the scalar curvature S of the corresponding two-dimensional surfaces as the quotient of functions$\{\phi^n \cosh m\phi\,, \phi^n \sinh m\phi\}$, and we derive the necessary and sufficient conditions for the coefficients to vanish identically. Finally an example is given to show three-dimensional surfaces with constant scalar curvature.

KEYWORDS: Catenary Surface, Equiform Motion, Scalar Curvature

1 INTRODUCTION

In physics and geometry, a catenary is a curve that an idealized hanging chain or cable assumes under its own weight when supported only at its ends. The curve has a U-like shape, superficially similar in appearance to a parabola, but it is not a parabola: it is a (scaled, rotated) graph of the hyperbolic cosine. The curve appears in the design of certain types of arches and as a cross section of the catenoid, the shape assumed by a soap film bounded by two parallel circular rings. Mathematically, the catenary curve is the graph of the hyperbolic cosine function. The surface of revolution of the catenary curve, the catenoid, is a minimal surface, specifically a minimal surface of revolution. The mathematical properties of the catenary curve were first studied by Robert Hooke in the 1670s, and its equation was derived by Leibniz, Huygens and Johann Bernoulli in 1691. Catenaries and related curves are used in architecture and engineering, in the design of bridges and arches, so that forces do not result in bending moments. In the offshore oil and gas industry, "catenary" refers to a steel catenary riser, a pipeline suspended between a production platform and the seabed that adopts an approximate catenary shape.

An equiform transformation in the n-dimensional Euclidean space $\mathcal{R}^n$ is an affine transformation whose linear part is composed by an orthogonal transformation and a homothetical transformation {3}-{10}. Such an equiform transformation maps points $\chi \in \mathcal{R}^n$ according to the rule

$$\boldsymbol{\chi \to sAx + d, A \in S(n), s \in \mathcal{R}^+, d \in \mathcal{R}^n} \qquad \textbf{(1)}$$

The number s is called the scaling factor. An equiform motion is determined if the parameters of (1}), including s, are given as functions of a time parameter t. Then an unruffled one-parameter equiform motion moves a point x via X(t)=s(t) A(t)x(t)+d(t). The kinematics corresponding to this transformation group is called similarity kinematics, see [1, 5].

In this paper, we study the scalar curvature of two-dimensional surfaces foliated by an equiform motion of a catenary curvec_0. Under a one-parameter an equiform motion of moving space $\Sigma^0$with respect to fixed space Σ. Suppose that $c_0 \subset \Sigma^0$ which is moved according to an equiform motion. The point paths of the catenary curve generate 2-dimensional surfaceχ, containing the position of the starting catenary curve. At any moment, the infinitesimal transformations of the motion will map the points of the catenary curve c_0 into the velocity vectors whose end points will form an affine image of c_0 that will be in general catenary curve in the moving spaceΣ. Both curves are planar and therefore, they span a subspace ω of$\mathbf{E}^n$, withdim $(\omega) \leq 5$. This if the reason why we restrict our considerations to dimension $n = 5$.

Let $X(\phi)$be a parametrization of $c_0$and $X(t,\phi)$ the definitive 3-surfaces foliated by the equiform motion. When we assign our study to the properties the motion for the limit case$\rightarrow 0$. A first option is when approximating $X(t,\phi)$ by the first derivative of the trajectories. The purpose of this work is to determine the two-dimensional surfaces acquired by the equiform motion of a catenary, where scalar curvature S is constant. The proof of our results comprises candid computations of the scalar curvature S of the surface $X(t,\phi)$. As we shall discuss, equation $\mathrm{S} = constant$. Furthermore, in this case, $\mathrm{S} = 0$ we show the depiction of the motion of such 2-surface giving the equations that define the kinematic geometry. We shall confer an example of such surfaces.

2 REPRESENTATION OF THE MOTION

Let c_0 be a unit catenary in the starting $x_1x_2 -$ plane of the moving space Σ^0 centered at the origin that represented by

$$X(\phi) = (\phi, \cosh\phi, 0,0,0)^T, \phi \in \mathcal{R}.$$

Under a one-parameter equiform motion of c_0 in the moving space, Σ^0 with respect to fixed spaceΣ, the position of a point $X(\phi) \in \Sigma^0$ at time t can be represented in the fixed system as

$$\chi(\mathrm{t},\phi) = \mathrm{s(t)A(t)}\chi(\phi) + \mathrm{d(t)}, \mathrm{t} \in \mathrm{I} \subset \mathcal{R}, \phi \in \mathcal{R}, \qquad \textbf{(2)}$$

Where $s(t)$ denotes the scaling factor of the moving system, $A(t) = \left(a_{ij}(t)\right), 1 \leq i,j \leq 5$ is an orthogonal matrix and $d(t) = \left(b_1(t), b_2(t), b_3(t), b_4(t), b_5(t)\right)^{\mathrm{T}}$ describes the position of the origin Σ^0 at the time t. For varying t and fixed $X(\phi), X(t,\phi)$ gives a parametric representation of the path (or trajectory) of (ϕ). Moreover, we assume that all involved functions are of class C^1. Expanding the two-dimensional surfaces giving by (2) using the Taylor's expansion up to first order, then we have

$$X(t,\phi) = \{s(0)A(0) + [\dot{\mathrm{s}}(0)\mathrm{A}(0) + \mathrm{s}(0)\dot{\mathrm{A}}(0)]\mathrm{t}\}\chi(\phi) + \mathrm{d}(0) + \mathrm{t}\dot{\mathrm{d}}(0)$$

Where $(.)$ indicates the differentiation with regard to t.

As an equiform motion has an invariant point, we can suppose that the moving frame Σ^0 and the steady frame Σ correspond at the zero position t=0. Then we have

$$A(0) = \mathrm{I}, s(0) = 1 \ and \ d(0) = 0.$$

Thus

$$X(t,\phi) = [I + (s'I + \Omega)t]X(\phi) + td',$$

Where $\Omega = \dot{A}(0) = (\omega_\kappa), 1 \le \mathrm{K} \le 10$, is a skew-symmetric matrix? Throughout this paper, all values of s, b_i and their derivatives are computed at $t = 0$ and for simplicity, we write s' and b_i' instead of $\dot{s}(0)$and $\dot{b}_i(0)$ respectively. In these frames, the representation of $X(t,\phi)$ is given by

$$\begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_4 \\ \chi_5 \end{pmatrix}(t,\phi) = \begin{pmatrix} 1+s't & t\omega_1 & t\omega_2 & t\omega_3 & t\omega_4 \\ -t\omega_1 & 1+s't & t\omega_5 & t\omega_6 & t\omega_7 \\ -t\omega_2 & -t\omega_5 & 1+s't & t\omega_8 & t\omega_9 \\ -t\omega_3 & -t\omega_6 & -t\omega_8 & 1+s't & t\omega_{10} \\ -t\omega_4 & -t\omega_7 & -t\omega_9 & -t\omega_{10} & 1+s't \end{pmatrix}\begin{pmatrix} \phi \\ \cosh\phi \\ 0 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} b_1' \\ b_2' \\ b_3' \\ b_4' \\ b_5' \end{pmatrix},$$

Or in the equivalent form

$$\begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_4 \\ \chi_5 \end{pmatrix}(t,\phi) = \phi\begin{pmatrix} 1+s't \\ -t\omega_1 \\ -t\omega_2 \\ -t\omega_3 \\ -t\omega_4 \end{pmatrix} + \cosh\phi\begin{pmatrix} -t\omega_1 \\ 1+s't \\ -t\omega_3 \\ -t\omega_4 \\ -t\omega_5 \end{pmatrix} + t\begin{pmatrix} b_1' \\ b_2' \\ b_3' \\ b_4' \\ b_5' \end{pmatrix}. \qquad \textbf{(3)}$$

For any stationary fixed t in the up expression (3), we generally get a catenary-shaped curve centered at the point $t(b_1', b_2', b_3', b_4', b_5')$ subject to the following conditions

$$\omega_2\omega_5 + \omega_3\omega_6 + \omega_4\omega_7 = 0, \qquad \textbf{(4-}\boldsymbol{i}\textbf{)}$$

$$\omega_2^2 + \omega_3^2 + \omega_4^2 = \omega_5^2 + \omega_6^2 + \omega_7^2. \qquad \textbf{(4-}\boldsymbol{ii}\textbf{)}$$

3 COMPUTATION TECHNIQUE OF SCALAR CURVATURE

In this section, we compute the scalar curvature S of the two-dimensional surfaces(t,ϕ). The proof of our results involves explicit computations of the scalar curvature S of the surface(t,ϕ). As we shall see, the equation $S = const.$ reduces to an expression that can be written as a linear combination of the functions$\{\phi^n \cosh m\phi, \phi^n \sinh m\phi\}$, namely$\sum_{n=0}^{4}\sum_{m=0}^{8}(E_{n,m}\phi^n \cosh m\phi + F_{n,m}\phi^n \sinh m\phi) = 0$, where $E_{n,m}$ *and* $F_{n,m}$two functions are depend on the variable$t$. In particular, the coefficients must vanish. The work then is to compute explicitly these coefficients $E_{n,m}$ *and* $F_{n,m}$ by successive manipulations. The authors were able to obtain the results using the symbolic program Mathematica 9 to check his work. The computer was used in each calculation several times, giving understandable expressions of the coefficients $E_{n,m}$ *and* $F_{n,m}$. See {3} for an example in a similar context. The tangent vectors to the parametric curves of $X(t,\phi)$ are

$$X_t(t,\phi) = (s'\mathrm{I} + \Omega)\chi(\phi) + d', X_\phi(t,\phi) = [\mathrm{I} + (s'\mathrm{I} + \Omega)t]\chi'(\phi).$$

Under the conditions (4), a straightforward computation leads to the coefficients of the first fundamental form defined by

$$g_{11} = \chi_t\chi_t^{\mathrm{T}}, g_{12} = \chi_\phi\chi_t^{\mathrm{T}}, g_{22} = \chi_\phi\chi_\phi^{\mathrm{T}}.$$

Where

$$g_{11} = \alpha + 2\alpha\beta\phi + \gamma\left(\phi^2 + \frac{1}{2}(1+\cosh 2\phi)\right) + 2\eta\cosh\phi, \qquad \mathbf{(5-\textit{i})}$$

$$g_{12} = b_1' + \beta\, t + (s' + \gamma\, t)\left(\phi + \frac{1}{2}\sinh 2\phi\right) + \omega_1\cosh\phi + (b_2' + \eta\, t - \omega_1\phi)\sinh\phi, \qquad \mathbf{(5-\textit{ii})}$$

$$g_{22} = \left(\frac{1}{2} + s't + \frac{1}{2}\gamma t^2\right)(1+\cosh 2\phi). (5-iii)$$

And

$$\alpha = \sum_{i=1}^{5} b_i'^2, \qquad \mathbf{(6-\textit{i})}$$

$$\beta = -(-s'b_1' + \omega_1 b_2' + \omega_2 b_3' + \omega_3 b_4' + \omega_4 b_5'), \qquad \mathbf{(6-\textit{ii})}$$

$$\eta = -(-s'b_2' - \omega_1 b_1' + \omega_5 b_3' + \omega_6 b_4' + \omega_7 b_5'), \qquad \mathbf{(6-\textit{iii})}$$

$$\gamma = s'^2 + \sum_{i=1}^{4}\omega_i'^2, \qquad \mathbf{(6-\textit{iv})}$$

The Christoffel symbols of the second kind are defined by

$$\Gamma_{ij}^{k} = \frac{1}{2}\sum_{m=1}^{2} g^{km}\left(\frac{\partial g_{im}}{\partial x_j} + \frac{\partial g_{jm}}{\partial x_i} + \frac{\partial g_{im}}{\partial x_m}\right).$$

Where $x_i \in \{t,\phi\}, \{i,j,k\}$ are indices that take the values 1 or 2 and (g^{lm}) is the inverse matrix of(g_{ij}). From here, the scalar curvature of the surface $X(t,\phi)$ is defined by (5)

$$S = \sum_{i,j,l=1}^{2} g^{ij}\left[\frac{\partial\Gamma_{ij}^{l}}{\partial x_l} - \frac{\partial\Gamma_{il}^{l}}{\partial x_j} + \sum_{m=1}^{2}\left(\Gamma_{ij}^{l}\Gamma_{lm}^{m} - \Gamma_{il}^{m}\Gamma_{jm}^{l}\right)\right].$$

Despite the computation of the scalar curvature S can be obtained, for example, by using the symbolic Mathematica programme, its expression is some hulking. At the zero position$t = 0$, the key in our proofs lies that we can write S as

$$S = \frac{P(\phi^n\cosh m\phi, \phi^n\sinh m\phi)}{Q(\phi^n\cosh m\phi, \phi^n\sinh m\phi)} = \frac{\sum_{m=0}^{2}\sum_{n=0}^{4}\left(A_{n,m}\phi^n\cosh m\phi + B_{n,m}\phi^n\sinh m\phi\right)}{\sum_{m=0}^{4}\sum_{n=0}^{8}\left(C_{n,m}\phi^n\cosh m\phi + D_{n,m}\phi^n\sinh m\phi\right)}. \qquad \mathbf{(7)}$$

The assumption of the constancy of the scalar curvature S implies that (7) can be converts into

$$S\,Q(\phi^n\cosh m\phi, \phi^n\sinh m\phi) - P(\phi^n\cosh m\phi, \phi^n\sinh m\phi) = 0. \qquad \mathbf{(8)}$$

Equation (8) indicates that if we write it as a linear combination of the functions $\{\phi^n\cosh m\phi, \phi^n\sinh m\phi\}$ namely, $\sum_{m=0}^{4}\sum_{n=0}^{8}\left(E_{n,m}\phi^n\cosh m\phi + F_{n,m}\phi^n\sinh m\phi\right) = 0$, the corresponding coefficients must vanish. Then we describe all two-dimensional surfaces with constant scalar curvature generated by equiform motion of catenary curve.

4 VANISHING SCALAR CURVATURE OF TWO-DIMENSIONAL SURFACES

Throughout this section, we shall assume that the two-dimensional surfaces $X(t,\phi)$ has zero scalar curvature ($S = 0$). From (7), we have

$$P(\phi^n\cosh m\phi, \phi^n\sinh m\phi) = \sum_{m=0}^{2}\sum_{n=0}^{4}\left(A_{n,m}\phi^n\cosh m\phi + B_{n,m}\phi^n\sinh m\phi\right) = 0 \qquad \mathbf{(9-\textit{i})}$$

$$Q(\phi^n \cosh m\phi, \phi^n \sinh m\phi) = \sum_{m=0}^{4}\sum_{n=0}^{8}\left(C_{n,m}\phi^n \cosh m\phi + D_{n,m}\phi^n \sinh m\phi\right) \neq 0 \qquad \textbf{(9}-\boldsymbol{ii}\textbf{)}$$

Then the work consists in the explicit computations of the coefficients $A_{n,m}$ and $B_{n,m}$. We distinguish the case that fill all possible cases, but we discuss this case (S = 0) under this condition $s' \neq 0$ and $b_1' b_2' \neq 0$. By solving the equation (9), we have $\beta = s'b_1' - \omega_1 b_2', \eta = s'b_2' + \omega_1 b_1'\ and\ \gamma = s'^2 + \omega_1'^2$. Then all coefficients $A_{n,m} = b_{n,m} = 0$ for all $0 \leq n \leq 2, 0 \leq m \leq 4$ vanish identically. Also, the coefficients $C_{n,m}\ and\ D_{n,m} \neq 0\ for\ 0 \leq n \leq 4, 0 \leq m \leq 8$. for example the coefficient $(C_{0,8})$ is giving by $\left[\frac{1}{32}\left(\frac{1}{2}S\,s'^4 - Ss'^2\gamma + \frac{1}{2}S\gamma^2\right)\right]$. That means the equation (9) holds $i.e., Q(\phi^n \cosh m\phi, \phi^n \sinh m\phi) = \sum_{m=0}^{4}\sum_{n=0}^{8}\left(C_{n,m}\phi^n \cosh m\phi + D_{n,m}\phi^n \sinh m\phi\right) \neq 0$ Then the scalar curvature S equal zero and from expression (5), we have the following conditions:

$$b_3'^2 + b_4'^2 + b_5'^2 = 0, \qquad \textbf{(10}-\boldsymbol{i}\textbf{)}$$

$$\omega_2 b_3' + \omega_3 b_4' + \omega_4 b_5' = 0, \qquad \textbf{(10}-\boldsymbol{ii}\textbf{)}$$

$$\omega_5 b_3' + \omega_6 b_4' + \omega_7 b_5' = 0, \qquad \textbf{(10}-\boldsymbol{iii}\textbf{)}$$

$$\omega_2'^2 + \omega_3'^2 + \omega_4'^2 = \omega_5'^2 + \omega_6'^2 + \omega_7'^2 = 0 \qquad \textbf{(10}-\boldsymbol{iv}\textbf{)}$$

This yields to:

$$\omega_i = 0\,; i = 2,3,4,\ldots,7\ and\ b_j' = 0\,; j = 3,4,5. \qquad \textbf{(11)}$$

We then conclude the following theorem.

Theorem 4.1

Let $X(t,\phi)$ be a two-dimensional surfaces acquired by the equiform motion of catenary curve c_0 and given by (3) under conditions giving by (4). Assume$b_1' b_2' \geq 0$, then the scalar curvature S vanishes identically on the surface if and only if the following conditions hold, on the surface if and only if the following two conditions:

1. $b_j' = 0, j = 3,4,5,$

2. $\omega_i' = 0, 2 \leq i \geq 7.$

Example 1

We assume $s(t) = e^{\mu t}$ such that $\mu \in \mathbb{R} - \{0\}$ and $d(t) = (t,t,0,0,0)^{\mathrm{T}}$.

Then $s' = \mu$ and $b_1' = b_2' = 1; b_3' = b_4' = b_5' = 0$. Now consider the following orthogonal matrix.

$$A(t) = \begin{pmatrix} \cos\mu\,t & \sin\mu\,t & 0 & 0 & 0 \\ -\sin\mu\,t & \cos\mu\,t & 0 & 0 & 0 \\ 0 & 0 & \cos^2\mu\,t & -\sin\mu\,t\cos\mu\,t & -\sin\mu t \\ 0 & 0 & \sin\mu\,t\cos\mu\,t & \cos^2\mu t & \sin\mu t \\ 0 & 0 & \sin\mu t\cos\mu t & -\sin\mu t & \cos^2\mu\,t \end{pmatrix},$$

Here, we have $\omega_1 = \omega_8 = \omega_{10} = \mu, \omega_9 = -\mu\ and\ \omega_k = 0\ for\ \sum_{k=2}^{7}$, Theorem 4.1 says that $S = 0$. In Figure 1, we display apiece of $X(t,\phi)$ of Example 1 in axonometric view point$Y(t,\phi)$. For this, the unit vectors $E_4 = (0,0,0,1,0)\ and\ E_5 = (0,0,0,0,1)$ are mapped onto the vectors $(1,1,0)\ and\ (0,1,1)$, respectively (4). Then

$$X(t,\phi)=t\begin{pmatrix}1\\1\\0\\0\\0\end{pmatrix}+\begin{pmatrix}1+\mu t\\-\mu t\\0\\0\\0\end{pmatrix}\phi+\begin{pmatrix}\mu t\\1+\mu t\\0\\0\\0\end{pmatrix}\cosh\phi$$

And

$$Y(t,\phi)=t\begin{pmatrix}1\\1\\0\end{pmatrix}+\begin{pmatrix}1+\mu t\\-\mu t\\0\end{pmatrix}\phi+\begin{pmatrix}\mu t\\1+\mu t\\0\end{pmatrix}\cosh\phi$$

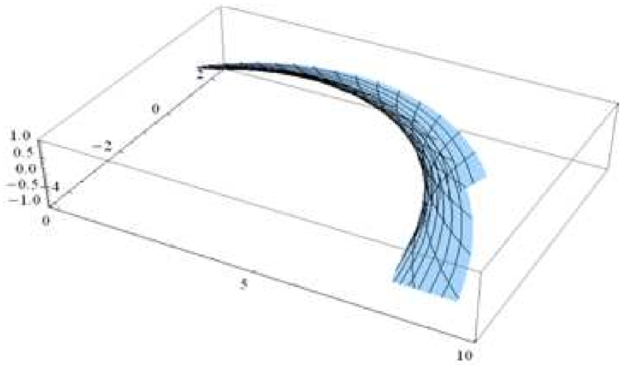

Figure 1: Corresponding Two-Dimensional Surfaces $X(t,\phi)$ with Equation (2) that Approximates

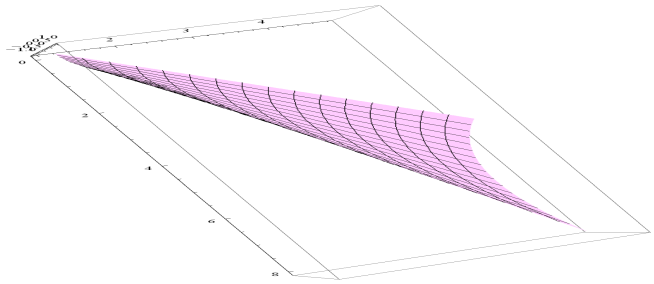

Figure 2: A Piece of Two-Dimensional Surfaces in Axonometric View $Y(t,\phi)$ with Zero Scalar Curvature

5 TWO-DIMENSIONAL OF SURFACES WITH S ≠ 0

In this section, we assume that the scalar curvature S of the two-dimensional surfaces $X(t,\phi)$ defined by (3) is a non-zero constant and $b_1 b_2 \neq 0$. the identity (8) writes then as

$$\sum_{m=0}^{4}\sum_{n=0}^{8}\left(E_{n,m}\phi^n\cosh m\phi+F_{n,m}\phi^n\sinh m\phi\right)=0 \quad \textbf{(12)}$$

Following the same scheme as in the case $S=0$ studied in section 4, we begin to compute the coefficients$E_{n,m}$ and $F_{n,m}$. Let us put $t=0$. the coefficients $F_{0,7}$ is giving by

$$F_{0,7}=\frac{1}{8}s'\omega_1 S(s'^2-\gamma)=0.$$

We have two possibilities:

- If $\omega_1=0$. the coefficient$F_{3,4}$ is given by $F_{3,4}=-s'^2\gamma\, S=0$. Implies that$\gamma=0$, since $s'\neq 0$ and from expression (6) we have $\gamma=0$ leads to a contradiction.

- If$\gamma = s'^2$. Then the coefficient of

$C_{4,3} = 2a^8 K s'^2(-s'^2 + \gamma) = 0$. We have, $\gamma = s'^2$ the coefficient of $E_{2,6} = \frac{1}{4} S s' {\omega_1}^2 = 0$. for $S \neq 0$ and $s' \neq 0$ and $\omega_1 \neq 0$ $then$ $\gamma = s'^2$ leads to contradiction. As epilogue of the above deduction, we conclude the following theorem**:**

Theorem 5.1

There are not two-dimensional kinematic surfaces obtained by the equiform motion of a catenary curve c_0 that given by (3) under conditions (4) whose scalar curvature S is a non-zero constant.

6 A DOMESTIC ISOMETRY BETWEEN 2-DIMENSIONAL SURFACES

In this section, we will research the presence of a local isometry between a two-dimensional surface in $\mathrm{E}^{\mathbf{5}}$ represented by $X(t,\phi)$ in (3) with constant scalar curvature and a two-dimensional surface in Euclidean three-space $\mathrm{E}^{\mathbf{3}}$**.** For more specifics see (4).

Now, we construct a two-dimensional surface $\bar{X}(t,\phi)$ in $\mathrm{E}^{\mathbf{3}}$ locally isometric $X(t,\phi)$ specified by (3). Where $X\colon \mathrm{U} \to S$ and $\bar{X}\colon \mathrm{U} \to \bar{S}$ defined in the same domain U such that $g_{11} = \overline{g_{11}}, g_{12} = \overline{g_{12}}$ and $g_{22} = \overline{g_{22}}$ in U. Then the map $\varphi = \bar{X} \circ X^{-1}\colon X(\mathrm{U}) \to \bar{S}$ is a local isometry. For this, we suppose that the initial catenary curve c_0 is the same that in $X(t,\phi)$**.** then $\bar{X}(t,\phi)$ writes as

$$\bar{\chi}(t,\phi) = \begin{pmatrix} 1+\bar{s}\,t & t\bar{\omega}_1 & t\bar{\omega}_2 \\ -t\bar{\omega}_1 & 1+\bar{s}\,t & t\bar{\omega}_3 \\ -t\bar{\omega}_2 & -t\bar{\omega}_3 & 1+\bar{s}\,t \end{pmatrix}\begin{pmatrix} \phi \\ \cosh\phi \\ 0 \end{pmatrix} + t\begin{pmatrix} \bar{b}_1 \\ \bar{b}_2 \\ \bar{b}_3 \end{pmatrix} \qquad \mathbf{(13)}$$

The computation of the first fundamental form of $\bar{X}(t,\phi)$ indicates to

$$\bar{g}_{11} = \bar{\alpha} + 2\bar{\beta}\phi + \bar{\gamma}\left(\phi^2 + \tfrac{1}{2}(1+\cosh 2\phi)\right) + 2\bar{\eta}\cosh\phi, \qquad (\mathbf{14-}\boldsymbol{i})$$

$$\bar{g}_{12} = \bar{b}_1' + \bar{\beta}\,t + (\bar{s}' + \bar{\gamma}\,t)\left(\phi + \tfrac{1}{2}\sinh 2\phi\right) + \bar{\omega}_1\cosh\phi + \left(\bar{b}_2' + \bar{\eta}\,t - \bar{\omega}_1\phi\right)\sinh\phi \qquad (\mathbf{14-}\boldsymbol{ii})$$

$$\bar{g}_{22} = \left(\tfrac{1}{2} + \bar{s}'t + \tfrac{1}{2}\bar{\gamma}t^2\right)(1+\cosh 2\phi). \qquad (\mathbf{14-}\boldsymbol{iii})$$

And

$$\bar{\alpha} = \bar{b}_1'^2 + \bar{b}_2'^2 + \bar{b}_3'^2, \qquad (\mathbf{15-}\boldsymbol{i})$$

$$\bar{\beta} = \bar{s}'\bar{b}_1' - \bar{\omega}_1\bar{b}_2' - \bar{\omega}_2\bar{b}_3', \qquad (\mathbf{15-}\boldsymbol{ii})$$

$$\bar{\eta} = \bar{s}'\bar{b}_2' + \bar{\omega}_1\bar{b}_1' - \bar{\omega}_3\bar{b}_3', \qquad (\mathbf{15-}\boldsymbol{iii})$$

$$\bar{\gamma} = \bar{s}'^2 + \bar{\omega}_1'^2 + \bar{\omega}_2'^2, \qquad (\mathbf{15-}\boldsymbol{iv})$$

As a consequence in this case calculated$\mathrm{E}^{\mathbf{5}}$, we have supposed that the original two axis of the catenary are orthogonal. This means$\bar{\omega}_2\bar{\omega}_3 = 0$. On the other hand, the first fundamental form of $X(t,\phi)$ was studied in (5). From X and$\bar{X}$, we have equations on the trigonometric functions $\sinh m\phi$ and $\cosh m\phi$.

The identities $g_{ij} = \overline{g_{ij}}$ imply

$s' = \bar{s}'^2, \omega_1 = \bar{\omega}_1, b_1' = \bar{b}_1', b_2' = \bar{b}_2'$

And $\alpha = \bar{\alpha}, \beta = \bar{\beta}, \gamma = \bar{\gamma}, \delta = \bar{\delta}, \eta = \bar{\eta}, \mu = \bar{\mu}, \lambda = \bar{\lambda}, \bar{\omega} = 0.$

Thus

$b_4'^2 + b_5'^2 = 0,$

$\bar{\omega}_2'^2 = \omega_2'^2 + \omega_3'^2 + \omega_4'^2,$

$\bar{\omega}_3'^2 = \omega_5'^2 + \omega_6'^2 + \omega_7'^2,$

$\bar{\omega}_2 \bar{b}_3' = \omega_2 b_3' + \omega_3 b_4' + \omega_4 b_5',$

$\bar{\omega}_3 \bar{b}_3' = \omega_5 b_3' + \omega_6 b_4' + \omega_7 b_5'.$

Theorem 6.1

Consider the two-dimensional surfaces in E^5 given by the parametrization $X(t,\phi)$ in(3) under condition(4) and with constant scalar curvature. Let $\bar{X}(t,\phi)$ be a kinematic surface in E^3 specified by (14). If the following equations:

$s' = \bar{s}'^2, \omega_1 = \bar{\omega}_1, b_1' = \bar{b}_1', b_2' = \bar{b}_2', b_4'^2 + b_5'^2 = 0.$

Then both surfaces $X(t,\phi)$ and $\bar $\bar{X}(t,\phi)$ are locally isometric. The Gaussian curvature of the surface $\bar{X}(t,\phi)$ in Euclidean space E^3 must vanish.

CONCLUSIONS

In this paper, we study the scalar curvature S of catenary curve in Equiform motion at E^5 and we concluded the cases of S in theorem (4.1) and (5.1).

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